How to Build a Short-Input Random Oracle *from* Public Random Permutations

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Eurocrypt 2026
Roma, Lazio, Italia

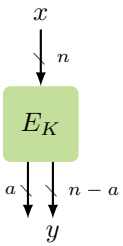
May 12, 2026

The Birthday Bound Problem

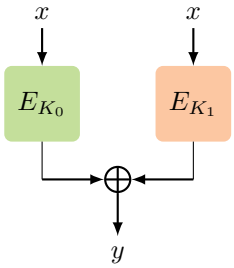
- Block ciphers: the most popular building blocks in symmetric cryptography
- Block size n for a fixed small n (e.g., 128) and key size κ
- $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$
- Standard security assumption: Pseudorandom Permutation (PRP)
- Commonly used to build Pseudorandom Functions (PRFs), e.g., in Counter Mode
- Counter Mode: $z \leftarrow E_K(x\|0)\|E_K(x\|1)\| \ldots E_K(x\|i) \ldots$
- x is a nonce, z is a random bitstream
- This is a secure PRF as long as E_K is not called more than $2^{n/2}$ times
- The $2^{n/2}$ term is the "birthday bound" PRP-PRF switching term
- Can get uncomfortable small when $n = 64$ (e.g., in PRINCE, Midori, SKINNY, Simon, PRESENT)

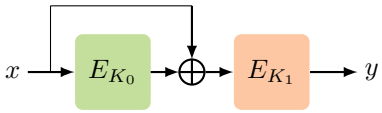
PRP-to-PRF

Truncation

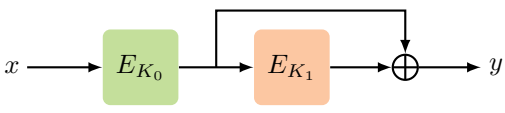


Sum Of Permutations





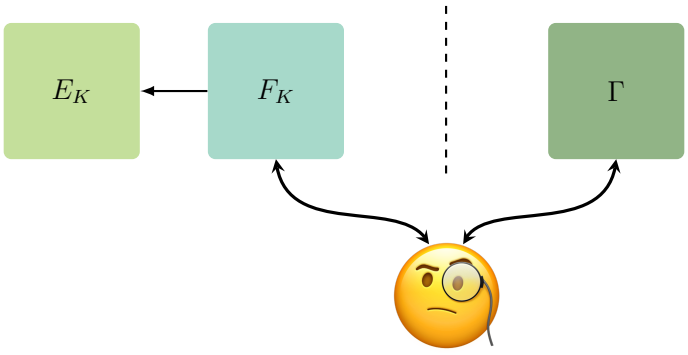
EDM



EDMD

Security Model: Indistinguishability

- F_k is a keyed function based on secret permutation Π, and Γ is a random function.



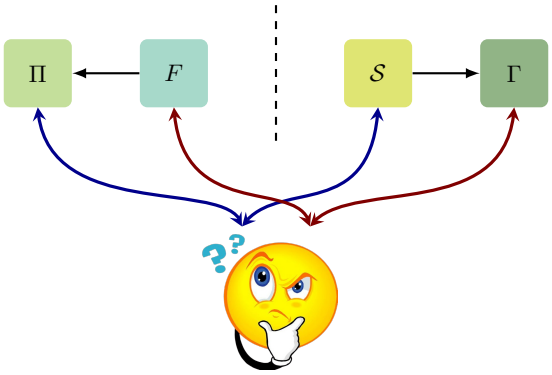
$$\mathsf{Adv}^{\mathsf{PRF}}_{\mathsf{F}}(\mathcal{A}) \coloneqq |\Pr_K[\mathcal{A}^{\mathsf{F}_K} \to 1] - \Pr_\Gamma[\mathcal{A}^{\Gamma} \to 1]|$$

Security in Keyless Setting

- PRF security of these schemes is well-understood by now
- However, PRF security depends heavily on the secrecy of the key
- Some level of security may also be desired when the key is leaked or even adversary-influenced
- An example is the notion of *committing security*
- Committing attack: find distinct $(K, x), (K', x')$ such that $F_K(x) = F_{K'}(x')$
- Comes in several variants, depending on application
- Encompassed by the stronger notion of *indifferentiability*
- This lets us build keyless Random Oracles from Random Permutations
- Here we focus on "short input" random oracles (an input of fixed width n)

Security Model: Indifferentiability

- F is a function based on a random permutation Π and Γ is a random oracle
- $\mathcal{S}$ is a simulator, which has access to Γ and imitates the public permutation Π



$$\mathsf{Adv}^{\mathsf{indiff}}_{F^{\Pi},\Gamma}(\mathcal{A}) := |\Pr_{\Pi}[\mathcal{A}^{F^{\Pi},\Pi} \to 1] - \Pr_{\Gamma}[\mathcal{A}^{\Gamma,\mathcal{S}^{\Gamma}} \to 1]|$$

Characterizing Short-I/O Random Oracles

- Π_0, Π_1 are two permutations over $\{0,1\}^n$
- $\alpha_i, \beta_i, \gamma_i \in \mathrm{GL}(\mathbf{F}_2^n) \cup \{0\}$ and $x \in \{0,1\}^n$:

$$\mathtt{F}^{\Pi_0,\Pi_1}(x) = \gamma_1 x \oplus \gamma_2 \Pi_0(\alpha_1 x) \oplus \gamma_3 \Pi_1(\beta_1 x \oplus \beta_2 \Pi_0(\alpha_1 x))$$

- $\mathtt{F}$ can be computed with the equation

$$\begin{pmatrix} u_0 \\ u_1 \\ \mathtt{F}(x) \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \beta_2 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \begin{pmatrix} x \\ \Pi_0(u_0) \\ \Pi_1(u_1) \end{pmatrix}$$

- We can assume $\gamma_1 = 0$, since the adversary can remove the effect of $\gamma_1 x$

Observations: Some Trivial Attacks

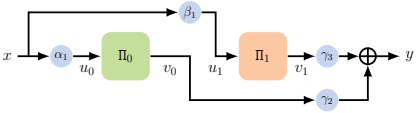
$$\mathtt{F}^{\Pi_0,\Pi_1}(x) = \gamma_2\Pi_0(\alpha_1 x) \oplus \gamma_3\Pi_1(\beta_1 x \oplus \beta_2\Pi_0(\alpha_1 x))$$

$$A = \begin{pmatrix} \alpha_1 & 0 & 0 \\ \beta_1 & \beta_2 & 0 \\ 0 & \gamma_2 & \gamma_3 \end{pmatrix}$$

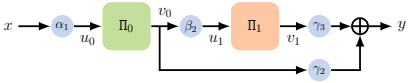
- When A contains either a zero row or a zero column, the influence of either the input x or one of Π_0 or Π_1 is lost, leading to easy attacks
- When $\beta_1 = \gamma_2 = 0$, $\mathtt{F}(x) = \gamma_3\Pi_1(\beta_2\Pi_0(\alpha_1 x))$, leading to the following attack:
  - Choose x
  - $z \leftarrow \mathtt{F}(\alpha_1^{-1}x)$
  - $v \leftarrow \Pi_1^{-1}(\gamma_3^{-1}z)$
  - $u \leftarrow \Pi_0^{-1}(\beta_2^{-1}v)$
  - Check whether the output equals x
  - In ideal world simulator cannot guess x after seeing z and v

Remaining Possible Constructions

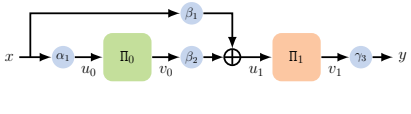
$$\mathrm{F}^{\Pi_0,\Pi_1}(x) = \gamma_2\Pi_0(\alpha_1 x) \oplus \gamma_3\Pi_1(\beta_1 x \oplus \beta_2\Pi_0(\alpha_1 x))$$



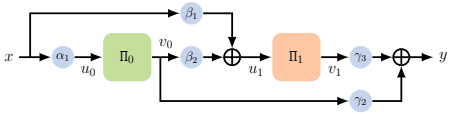
$A_{\texttt{SOP}}$ $(\beta_2 = 0, \beta_1, \gamma_2 \neq 0)$



$A_{\texttt{EDMD}}$ $(\beta_1 = 0, \beta_2, \gamma_2 \neq 0)$



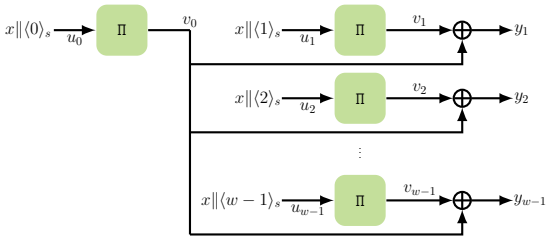
$A_{\texttt{EDM}}$ $(\gamma_2 = 0, \beta_1, \beta_2 \neq 0)$



$A_{\texttt{DMDM}}$ $(\beta_1, \beta_2, \gamma_2 \neq 0)$

Overview of analysed constructions

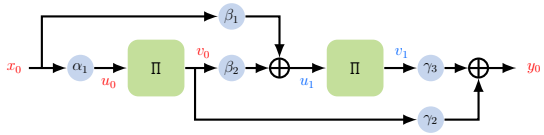
- We analyze these four constructions under three different assumptions on Π_0 and Π_1:
  - $\Pi_1 = \Pi_0$
  - $\Pi_1 = \Pi_0^{-1}$
  - Π_0 and Π_1 are independent
- In addition we look at the short-input long-output random oracle $\mathtt{XORP}^{\Pi}_{w}$:



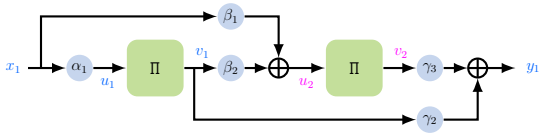
Summary of Analysis

- None of these is indifferentiable when $\Pi_1 = \Pi_0$
- None of these is indifferentiable when $\Pi_1 = \Pi_0^{-1}$
- Both these cases are broken with a new *chaining attack*
- When Π_0 and Π_1 are independent, we have positive results:
  - SOP and EDMD are indifferentiable up to $2^{2n/3}$ queries
  - EDM and DMDM are indifferentiable up to $2^{n/2}$ queries (with matching attacks)
- Further we show that $\mathtt{XORP}_w$ is indifferentiable up to $2^{2n/3}/w^{4/3}$ queries

Creating a Chain when $\Pi_1 = \Pi_0 = \Pi$



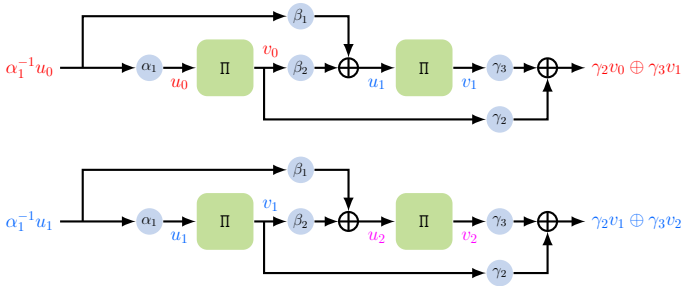
- Choose v_0
- Query $u_0 \leftarrow \Pi^{-1}(v_0)$
- $x_0 \leftarrow \alpha_1^{-1} u_0$
- Query $y_0 \leftarrow \mathtt{F}(x_0)$
- $u_1 \leftarrow \beta_1 \alpha_1^{-1} u_0 \oplus \beta_2 v_0$
- $v_1 \leftarrow \gamma_3^{-1}(y_0 \oplus \gamma_2 v_0)$



- We already have v_1 and u_1
- $x_1 \leftarrow \alpha_1^{-1} u_1$
- Query $y_1 \leftarrow \mathtt{F}(x_1)$
- $u_2 \leftarrow \beta_1 \alpha_1^{-1} u_1 \oplus \beta_2 v_1$
- $v_2 \leftarrow \gamma_3^{-1}(y_1 \oplus \gamma_2 v_1)$

Creating a Chain when $\Pi_1 = \Pi_0 = \Pi$

- $((u_0, v_0), (u_1, v_1), (u_2, v_2) \ldots, (u_m, v_m))$ is said to be an m-*chain* if $v_i = \Pi(u_i)$ and $u_{i+1} = \beta_1 \alpha_1^{-1} u_i \oplus \beta_2 v_i$ for all $i \in [m-1]$

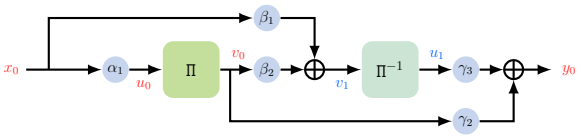


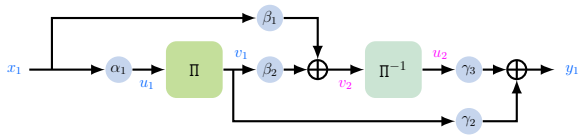
- In the real world, $\mathtt{F}(\alpha_1^{-1}u_0) \oplus \cdots \oplus \mathtt{F}(\alpha_1^{-1}u_i) = \gamma_2 v_0 \oplus \bigoplus_{j=1}^{i} (\gamma_2 \oplus \gamma_3) v_j \oplus \gamma_3 v_{i+1}$
- Efficient simulator cannot maintain this relation for large m

Mounting an Attack using a Chain

- Creating and testing a chain requires only two simulator queries
- Simulator's expected complexity is polynomial in the numer of simulator queries
- A simulator receiving two queries can make $O(2)$ queries to Γ
- For some m_{MAX}, it can keep track of a chain of length at most m_{MAX} (with high probability)
- An adversary testing a chain of length $m_{\mathsf{MAX}} + 1$ wins with high probability
- Thus for any simulator there is an adversary it cannot defeat

Creating a Chain when $\Pi_0 = \Pi, \Pi_1 = \Pi^{-1}$



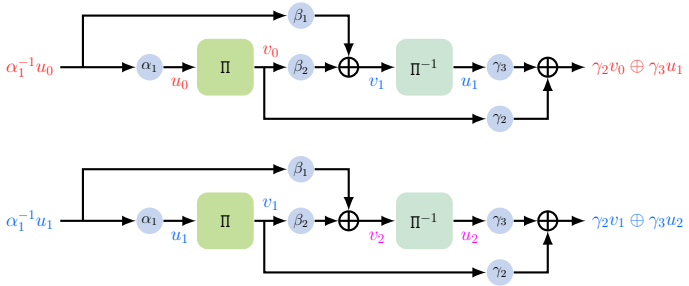


- Choose u_0
- Query $v_0 \leftarrow \Pi(u_0)$
- $x_0 \leftarrow \alpha_1^{-1} u_0$
- Query $y_0 \leftarrow \mathtt{F}(x_0)$
- $v_1 \leftarrow \beta_1 \alpha_1^{-1} u_0 \oplus \beta_2 v_0$
- $u_1 \leftarrow \gamma_3^{-1}(y_0 \oplus \gamma_2 v_0)$

- We already have u_1 and v_1
- $x_1 \leftarrow \alpha_1^{-1} u_1$
- Query $y_1 \leftarrow \mathtt{C}(x_1)$
- $v_2 \leftarrow \beta_1 \alpha_1^{-1} u_1 \oplus \beta_2 v_1$
- $u_2 \leftarrow \gamma_3^{-1}(y_1 \oplus \gamma_2 v_1)$

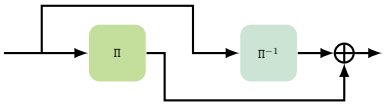
Creating a Chain when $\Pi_0 = \Pi, \Pi_1 = \Pi^{-1}$

- $((u_0, v_0), (u_1, v_1), (u_2, v_2) \ldots, (u_m, v_m))$ is said to be an m-chain if $v_i = \Pi(u_i)$ and $u_{i+1} = \gamma_3^{-1}(\mathtt{F}(\alpha^{-1}u_i) \oplus \gamma_2 v_i)$ for all $i \in [m-1]$



- In the real world, $\mathtt{F}(\alpha_1^{-1}u_0) \oplus \cdots \oplus \mathtt{F}(\alpha_1^{-1}u_i) = \bigoplus_{j=0}^{i} \gamma_2 v_j \oplus \bigoplus_{j=0}^{i} \gamma_3 u_{j+1}$

An Interesting Corollary



- Dodis et al. proposed [DPP08, Lemma 4] a simulator for this construction
- They claimed a birthday-bound (strong) indifferentiability security with the proposed simulator
- However, our chaining attack invalidates this claimed result
- We show a concrete attack on the proposed simulator
- More generally, we show that no simulator can work for this construction

[DPP08] Dodis, Y., Pietrzak, K., Puniya, P.: A New Mode of Operation for Block Ciphers and Length-Preserving MACs. EUROCRYPT 2008

Applications

- Indifferentiability can be translated to committing security by considering a multi-instance setting
- Constructions using hash function are often proved secure in the Random Oracle model
- Indifferentiability results justify instantiating a Random Oracle with a concrete construction
- Short Input ROs are used in the Fiat-Shamir transform to make public-coin proofs non-interactive
- MPC-in-the-head protocols can use $\texttt{XORP}_w$ instead of GGM trees for better efficiency
- Many proofs of public-key protocols rely on Short Input ROs
- Our overall impression: having a suite of Short Input ROs should come in handy

Thank You





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