Efficient Variants of TNT with BBB Security

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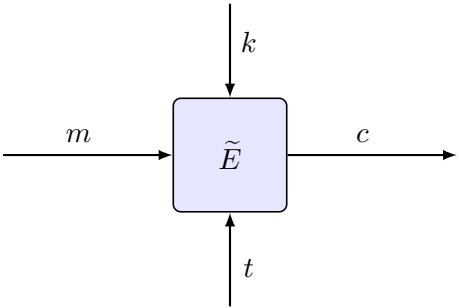
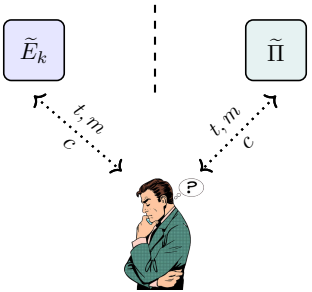
Tweakable Block Cipher



- A family of permutations indexed by secret key and public tweak
- $\widetilde{E}\colon \mathcal{M} \times \mathcal{K} \times \mathcal{T} \to \mathcal{C}$, where $\mathcal{M} \coloneqq \{0,1\}^n, \mathcal{K} \coloneqq \{0,1\}^k$, $\mathcal{T} \coloneqq \{0,1\}^t$ and $\mathcal{C} \coloneqq \{0,1\}^n$
- For a fixed key, tweak pair (k,t), $\widetilde{E}_{k,t}$ is a permutation over $\{0,1\}^n$
- TBCs have found diverse applications in designing of AE schemes, MACs, PRFs, Wide block encryption modes

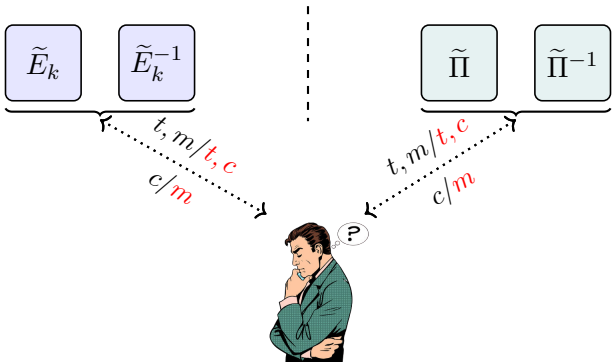
Security Notions of Tweakable Block Cipher: TPRP

- $\widetilde{E}\colon \mathcal{M} \times \mathcal{K} \times \mathcal{T} \to \mathcal{C}$ is a tweakable block cipher
- $\widetilde{\Pi}$ is a tweakable random permutation, a family of independent random permutations parameterized by tweak t



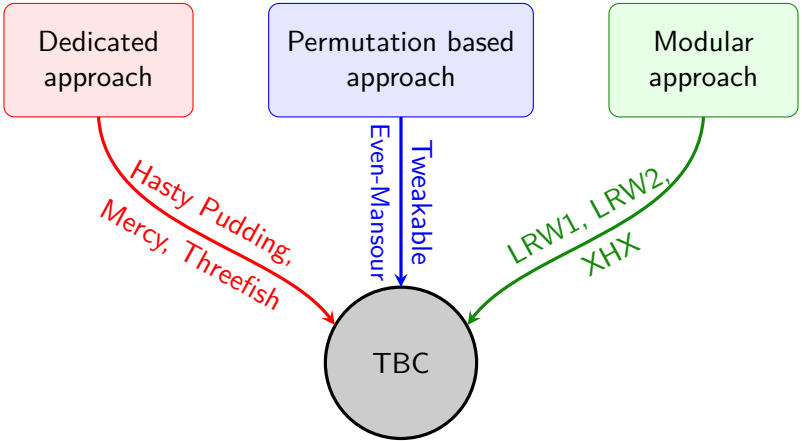
$$\mathsf{Adv}^{\mathsf{TPRP}}_{\mathcal{A},\widetilde{E}}(q) \coloneqq |\Pr[k \xleftarrow{\$} \mathcal{K}\colon \mathcal{A}^{\widetilde{E}_k(\cdot,\cdot)} \to 1] - \Pr[\widetilde{\Pi} \xleftarrow{\$} \mathsf{TPerm}(n)\colon \mathcal{A}^{\widetilde{\Pi}(\cdot,\cdot)} \to 1]|$$

Security Notions of Tweakable Block Cipher: STPRP



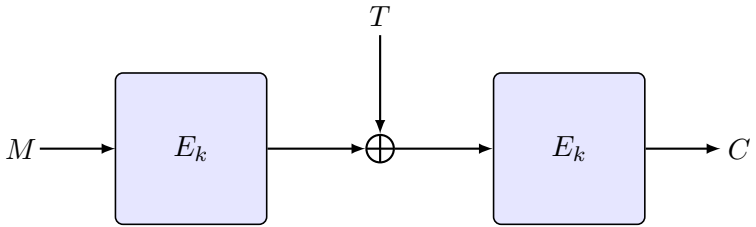
$$\mathsf{Adv}^{\mathsf{STPRP}}_{\mathcal{A},\widetilde{E}}(q) \coloneqq |\Pr[k \xleftarrow{\$} \mathcal{K} \colon \mathcal{A}^{\widetilde{E}_k(\cdot,\cdot),\widetilde{E}_k^{-1}(\cdot,\cdot)} \to 1] - \Pr[\widetilde{\Pi} \xleftarrow{\$} \mathsf{TPerm}(n) \colon \mathcal{A}^{\widetilde{\Pi}(\cdot,\cdot),\widetilde{\Pi}^{-1}(\cdot,\cdot)} \to 1]|$$

Design Approaches of TBC



In this work, we are focusing on Modular approach

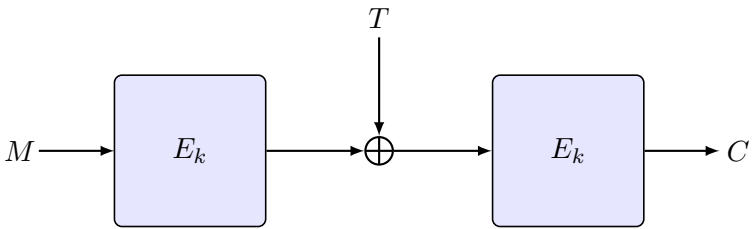
Modular Approach: Designing TBC from BC



LRW1 Construction

- LRW1 construction was proposed by Liskov et al. at CRYPTO'02
- It achieves Tight CPA security upto $2^{n/2}$ queries, assuming E_k is n-bit secure PRP

Modular Approach: Designing TBC from BC



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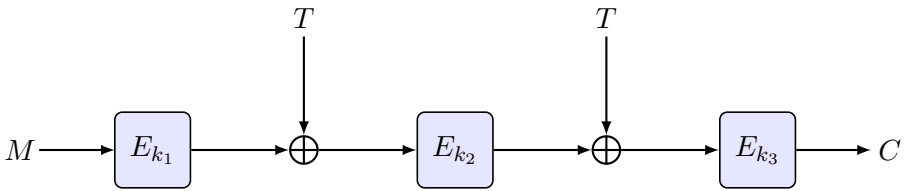
LRW1 is Birthday Bound secure

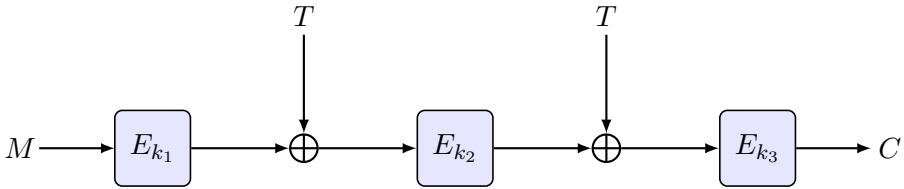
Seeking BBB secure TBC (Cascaded LRW1)



CLRW1^3 (TNT) Construction

- Proposed by Bao et al. at EUROCRYPT'20
- [Bao et al., EUROCRYPT'20]: CCA security upto $2^{2n/3}$ queries
- [Guo et al., ASIACRYPT'20]: Tight CPA security upto $2^{3n/4}$ queries
- [Jha et al., EUROCRYPT'24]: Tight CCA security upto $2^{n/2}$ queries

Seeking BBB secure TBC (Cascaded LRW1)



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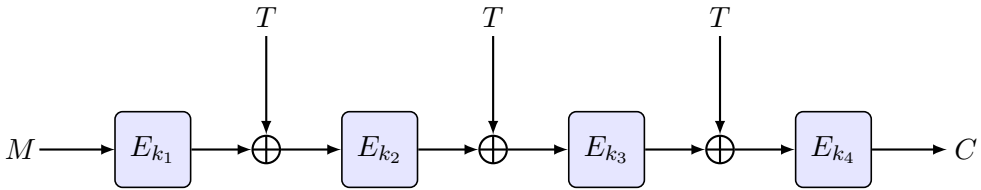
TNT now achieves tight Bithday-Bound security

The search begins anew



CLRW1^4 Construction

- Proposed by Datta et al. at ToSC Volume 2023, Issue 4
- [Datta et al., ToSC'23(4)]: CCA security upto $2^{3n/4}$ queries

Efficient Variants of TNT with BBB Security

Can we fix TNT efficiently with BBB security?

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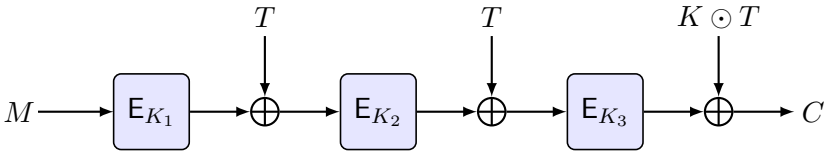
Our Contributions :

- We have proposed two efficient (compared to CLRW1[4]) variants of TNT construction
  - b-TNT1
  - b-TNT2
- We have shown that both constructions achieve CCA security upto $2^{3n/4}$ queries
- We have experimentally verified that both b-TNT1 and b-TNT2 perform better than CLRW1[4] in terms of throughput

b-TNT1 Construction

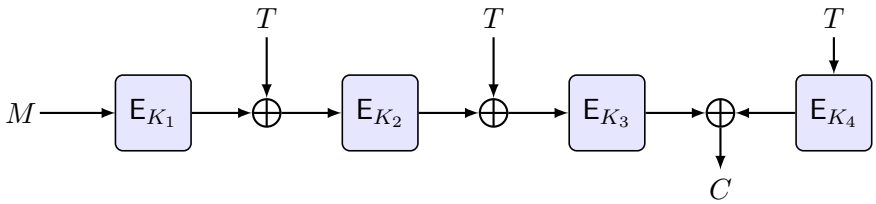


b-TNT1 construction (3 BC calls and 1 field multiplication)

Security Result

$$\mathbf{Adv}^{\mathrm{STPRP}}_{\mathsf{b\text{-}TNT1}}(\mathsf{A}) \leq 3\mathbf{Adv}^{\mathrm{SPRP}}_{\mathsf{E}}(\mathsf{A}') + \frac{3q^2}{2^{2n}} + \frac{5q^{4/3}}{2^n} + \frac{45q^4}{2^{3n}} + \frac{1}{2^n}$$

b-TNT2 Construction

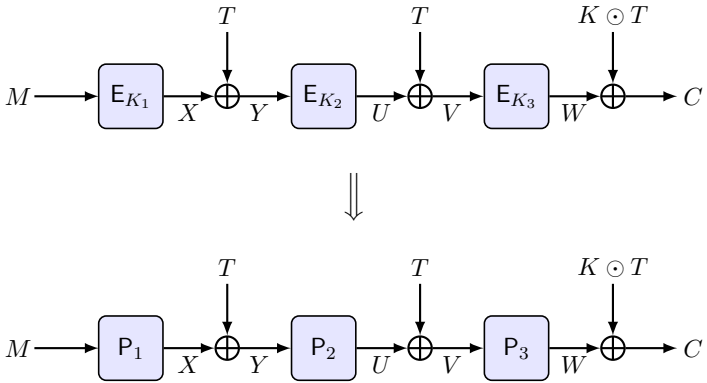


b-TNT2 construction (4 BC calls)

Security Result

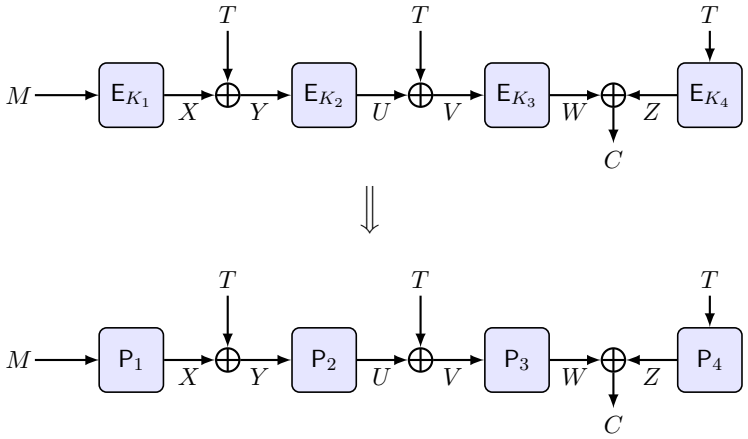
$$\mathbf{Adv}^{\mathrm{STPRP}}_{\mathsf{b\text{-}TNT2}}(\mathsf{A}) \leq 3\mathbf{Adv}^{\mathrm{SPRP}}_{\mathsf{E}}(\mathsf{A}') + \mathbf{Adv}^{\mathrm{PRP}}_{\mathsf{E}}(\mathsf{B}) + \frac{4q^2}{2^{2n}} + \frac{6q^{4/3}}{2^n} + \frac{53q^4}{2^{3n}}.$$

Proof Sketch: Replacing BCs with random permutations (b-TNT1)



- At the cost of the strong pseudorandom permutation advantage of the underlying BC.

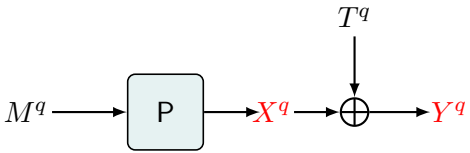
Proof Sketch: Replacing BCs with random permutations (b-TNT2)

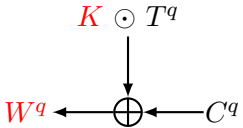


- At the cost of the sprp advantage and prp advantage (for E_{K_4}) of the underlying BC.

Proof Sketch: Releasing intermediate variables (b-TNT1)

- In the real world the original intermediate values $(X^q, Y^q, U^q, V^q, W^q, K)$ will be released
- In the ideal world the intermediate values will be sampled accordingly:
  - K will be sampled as a dummy key from the key space

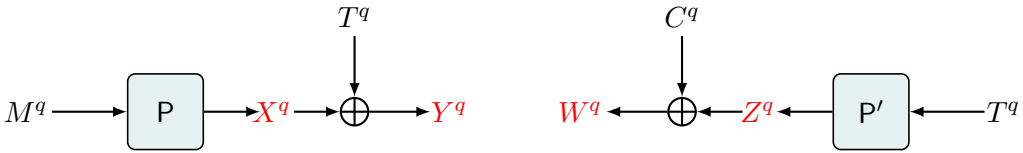




(U^q, V^q) is yet to be sampled

Proof Sketch: Releasing intermediate variables (b-TNT2)

- In the real world the original intermediate values $(X^q, Y^q, U^q, V^q, W^q, Z^q)$ will be released
- In the ideal world the intermediate values will be sampled accordingly:



(U^q, V^q) is yet to be sampled

Proof Sketch: Bad partial transcript

Partial transcript: $X^q, Y^q, W^q, K(\text{b-TNT1})/Z^q(\text{b-TNT2})$

- The partial transcript is called Bad if one of the following holds:
  1. Bad_K: $K = 0^n$ (This condition is only for b-TNT1).
  2. Bad_1 (cycle of length 2): $\exists i, j \in [q]$ such that the following holds: $Y_i = Y_j, W_i = W_j$.
  3. Bad_2: $|\{(i,j) \in [q]^2 : i \neq j, Y_i = Y_j\}| \geq q^{2/3}$.
  4. Bad_3: $|\{(i,j) \in [q]^2 : i \neq j, W_i = W_j\}| \geq q^{2/3}$.
  5. Bad_4 (Y-W-Y path of length 4): $\exists i, j, k, l \in [q]$ such that the following holds: $Y_i = Y_j, W_j = W_k, Y_k = Y_l$.
  6. Bad_5 (W-Y-W path of length 4): $\exists i, j, k, l \in [q]$ such that the following holds: $W_i = W_j, Y_j = Y_k, W_k = W_l$.

Proof Sketch: Constructing transcript graph

- For a bad partial transcript, (U^q, V^q) will be sampled degenerately i.e., $U_i = V_i = 0$ $\forall i \in [q]$.

- For a good partial transcript construct an edge labeled bipartite graph $\mathcal{G} := \mathcal{G}(Y^q, W^q)$:

  - **Vertices:** $Y^q = \{Y_1, \ldots, Y_q\} \bigsqcup W^q = \{W_1, \ldots, W_q\}$

  - **Labeled edges:** $\{Y_i, W_i\} \in E$ with label T_i

  - For two distinct indices $i \neq j$, if $Y_i = Y_j$, then we merge the corresponding vertices. Similarly, for two distinct indices, if $W_i = W_j$, then we merge the corresponding vertices.

Proof Sketch: Structures of a good transcript graph

Properties of a good transcript graph

- Simple, contains no cycle
- Has no even length path with label sum 0
- Every path has a maximum length of 3
- Maximum component size can be $2q^{2/3}$

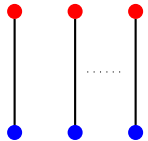


Figure: Type-1

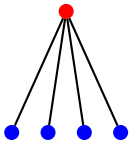


Figure: Type-2

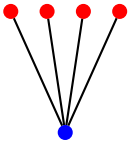


Figure: Type-3

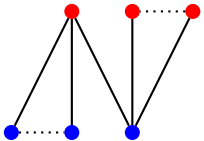


Figure: Type-4

Proof Sketch: Sampling (U^q, V^q) for good transcripts

- Consider $\mathcal{I} = \mathcal{I}_1 \sqcup \mathcal{I}_2 \sqcup \mathcal{I}_3$, where $\mathcal{I}_b = \{i \in [q] : (Y_i, W_i) \in \text{Type-}b\}$ for $b \in [4]$
- Consider $\mathcal{E} = \{U_i \oplus V_i = T_i : i \in \mathcal{I}\}$, where $U_i = U_j$ iff $Y_i = Y_j$ and $V_i = V_j$ iff $W_i = W_j$ for all $i \neq j \in [q]$
- Solution set of $\mathcal{E}$, $\mathcal{S} = \{(u^{\mathcal{I}}, v^{\mathcal{I}}) : u^{\mathcal{I}} \leftrightsquigarrow Y^{\mathcal{I}}, v^{\mathcal{I}} \leftrightsquigarrow W^{\mathcal{I}}, u^{\mathcal{I}} \oplus v^{\mathcal{I}} = T^{\mathcal{I}}\}$
- Now we sample (U^q, V^q) in the following way:
  - $(U^{\mathcal{I}}, V^{\mathcal{I}}) \xleftarrow{\$} \mathcal{S}$, i.e., it uniformly samples one valid solution from the set of all valid solutions
  - For Type-4 graph select (Y_i, W_i) such that the degree of both Y_i and W_i is at least 2. Then, we sample $U_i \xleftarrow{\$} \{0,1\}^n$ and set $V_i = U_i \oplus T_i$
  - Then for the other edges (Y_j, W_j) of Type-4 graph, either $Y_j = Y_i$, then $U_j = U_i$ and $V_j = U_j \oplus T_j$ or $W_j = W_i$, then $V_j = V_i$ and $U_j = V_j \oplus T_j$

Proof Sketch: Sampling induced bad events

- The sampling of the values (U^q, V^q) may lead to permutation incompatibility

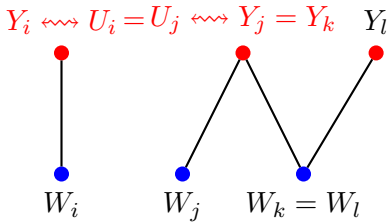


Figure: $\mathsf{Ucoll}_{1,4}$

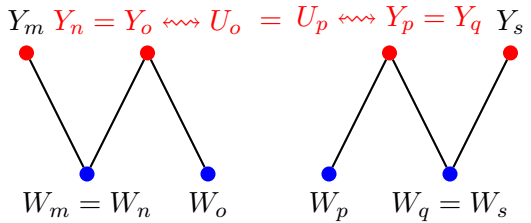


Figure: $\mathsf{Ucoll}_{4,4}$

- $\mathsf{Ucoll}_{\alpha\beta}$: $\exists i \in \mathcal{I}_\alpha$, $j \in \mathcal{I}_\beta$ such that $Y_i \neq Y_j$ and $U_i = U_j$, for $\alpha \in [4]$ and $\beta \in [\alpha, 4]$
- $\mathsf{Vcoll}_{\alpha\beta}$: $\exists i \in \mathcal{I}_\alpha$, $j \in \mathcal{I}_\beta$ such that $W_i \neq W_j$ and $V_i = V_j$, for $\alpha \in [4]$ and $\beta \in [\alpha, 4]$

$$\mathsf{Bad\text{-}samp} := \bigcup_{\substack{\alpha \in [4] \\ \beta \in [\alpha, 4]}} (\mathsf{Ucoll}_{\alpha,\beta} \cup \mathsf{Vcoll}_{\alpha,\beta})$$

Proof Sketch: Good transcript analysis

- Real World: Counted the number of calls of each permutations.
- Ideal World:
  - For graph of Type-1,2,3: Used Mirror Theory results for the tweakable random permutations developed by Jha and Nandi [JN, JoC'20]
  - For graph of Type-4: Counted the number of components
- Finally, using Expectation Method we derive the advantage bound of both the constructions.

Experimental Results

Cycles and cycles per byte for proposed constructions, constructions labeled with * also include the key schedule cost.

Construction	Cycles	Cycles per byte
CLRW1^4	184	11.5
b-TNT1	150	9.37
b-TNT2	164	10.25
CLRW1^{4*}	1719	107.44
b-TNT1$*$	1240	77.5
b-TNT2$*$	1645	102.81

- Unlike CLRW1^4, b-TNT1 requires three block cipher calls along with a field multiplication
- b-TNT2 requires four block cipher calls but its execution of block cipher calls can be pipelined which makes it efficient over CLRW1^4

Open Problems

- Is the proven security bound of b-TNT1 and b-TNT2 tight?
- Can we design any other efficient BBB secure TBC constructions?
- What about the multi user security of the constructions b-TNT1, b-TNT2 and CLRW1[4]?

Thank You!